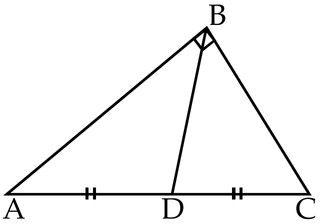
Q.P. SET CODE
B

MT - x

2018 ___ ___ 1100 - MT - x - MATHEMATICS (71) Geometry - SET - B (E)

Time : 2 Hours **Preliminary Model Answer Paper** **Max. Marks : 40**

A.1.(A)	**Solve ANY FOUR of the following :**	
(i)	If diagonals of a parallelogram are congruent, then it is a rectangle.	1
(ii)	If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is <u>180°</u>.	1
(iii)	In ΔABC, $\angle ABC = 90^{\circ}$ Seg BD is the median on hypotenuse AC.  $BD = \frac{1}{2} AC$ (Median drawn to the hypotenuse is half of it) $\therefore \quad 7 = \frac{1}{2} AC$ $\therefore \quad AC = 14$ cm	1
(iv)	A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent.	1
(v)	Equation of the Y – axis is X = 0	1
(vi)	$\frac{\tan 40^{\circ}}{\cot 50^{\circ}}$ $= \frac{\tan 40^{\circ}}{\tan(90-50)^{\circ}}$ $= \frac{\tan 40^{\circ}}{\tan 40^{\circ}}$ $= 1$	1

A.1.(B) Solve ANY TWO of the following :

(i) For a cone

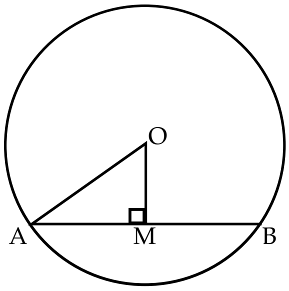
Area of the base = 1386 sq. cm Height (h) = 28 cm

Volume of a cone $= \pi r^2 h$ 1

$= \frac{1}{3} \times 1386 \times 28$ ½

Volume of a cone = 12936 cm^3 ½

(ii) A circle with centre O



chord AB = 24 cm

seg OM ⊥ chord AB

OM = 5 cm

AM $= \frac{1}{2}$ AB ½

[Perpendicular drawn from the centre to the chord, bisects the chord]

$= \frac{1}{2} \times 24$

AM = 12 cm ½

In ΔOMA, ∠OMA = 90°

$OA^2 = 5^2 + 12^2$ [Pythagoras theorem] ½

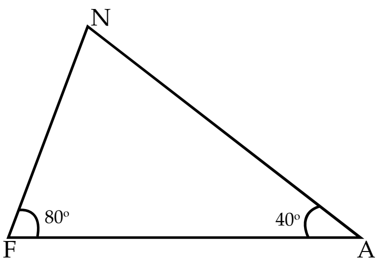
∴ $OA^2 = 25 + 144$

∴ $OA^2 = 169$

∴ OA = 13 cm

∴ Radius of the circle is 13 cm ½

(iii) In ΔFAN,



∠F = 80°

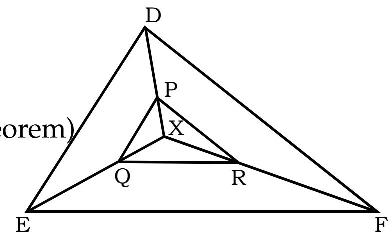
∠A = 40°

∴ ∠N = 60° [Remaining angle] ½

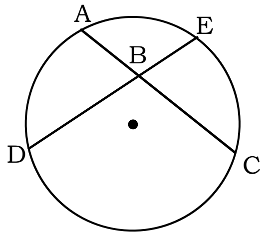
∴ ∠F > ∠N > ∠A ½

∴ AN > AF > FN [In a triangle, side opposite to greater angle is greater] ½

∴ Greatest side is AN and the smallest side is FN ½

A.2.(A)	**Select the correct alternative answer and write it :**	
(i)	(c) $4\sqrt{2}$	1
(ii)	(b) $\operatorname{cosec}^2 \theta - \sin^2 \theta = 1$	1
(iii)	(b) 25 cm	1
(iv)	(c) $\frac{1}{\sqrt{3}}$	1
A.2.(B)	**Solve ANY TWO of the following :**	
(i)	$B(k, -5) = (x_1, y_1)$ $C(1, 2) = (x_2, y_2)$	
	Slope of line BC $= \frac{y_2 - y_1}{x_2 - x_1}$	½
	$\therefore \quad 7 = \frac{2-(-5)}{1-k}$	½
	$\therefore \quad 7(1-k) = 2 + 5$ $\therefore \quad 7(1-k) = 7$	½
	$\therefore \quad 1 - k = \frac{7}{7}$ $\therefore \quad 1 - k = 1$ $\therefore \quad 1 - 1 = k$ $\therefore \quad k = 0$	½
(ii)	Proof : In XDE, PQ $\parallel$ DE ...(Given)	
		
	$\therefore \quad \frac{XP}{PD} = \frac{XQ}{QE}$ ...(i) (Basic proportionality theorem)	½
	In XEF, seg QR $\parallel$ side EF ... (Given)	
	$\therefore \quad \frac{XQ}{QE} = \frac{XR}{RF}$ ...(ii) (Basic proportionality theorem)	½
	$\therefore \quad \frac{XP}{PD} = \frac{XR}{RF}$ ...[From (i) and (ii)]	½
	$\therefore$ seg PR $\parallel$ side DF ...(Converse of Basic Proportionality theorem)	½

(iii) $m\angle ABE = \frac{1}{2}$ [m(arc DC) + m(arc AE)] ½



$\therefore$ $108 = \frac{1}{2}$ [m(arc DC) + 95] ½

$\therefore$ 216 = m(arc DC) + 95

$\therefore$ m(arc DC) = 216 – 95 ½

$\therefore$ $\boxed{\text{m(arc DC)} = 121}$ ½

A.3.(A) Carry out ANY TWO of the following activites :

(i) Given : In Δ ABC, $\angle$ ABC = 90°

To prove : $AC^2 = AB^2 + BC^2$

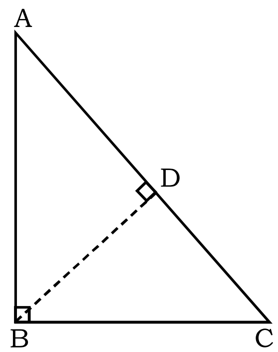
Construction : Draw seg BD $\perp$ hypotenuse AC. A – D – C

Proof : In ΔABC, $\angle$ ABC = 90° (Given)

seg BD $\perp$ hypotenuse AC (Construction)

$\therefore$ $\boxed{\Delta ABC} \sim \boxed{\Delta ADB} \sim \boxed{\Delta BDC}$ (Similarity of right angle triangle) 2

Δ ABC ~ Δ ADB



$\frac{AB}{\boxed{AB}} = \frac{\boxed{AC}}{AB}$

$\therefore$ $AB^2 = \boxed{AC \times AD}$ ...(i)

Δ ABC ~ Δ BDC

$\frac{BC}{\boxed{DC}} = \frac{\boxed{AC}}{BC}$ ...(ii)

$BC^2 = \boxed{AC \times DC}$ ...(ii)

Adding (i) and (ii),

$AB^2 + BC^2 = \boxed{AC \times AD + AC \times DC}$

$AB^2 + BC^2 = AC\ (AD + DC)$

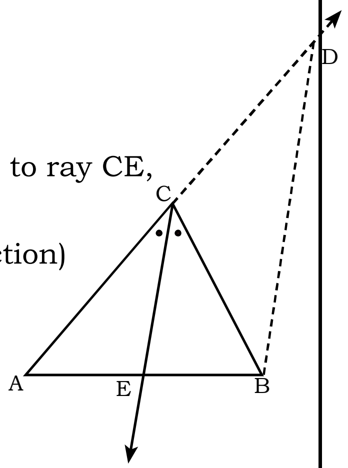
$AB^2 + BC = AC \times AC(AD + DC)$

$AB^2 + BC^2 = AC^2$

(ii) Given : In Δ ABC, seg CE bisects ∠ ACB

To prove : $\frac{AE}{EB} = \frac{AC}{CB}$

Construction : Through B, draw a line parallel to ray CE, Extend AC to intersect it at point D.



Proof : In Δ ABD, seg EC || seg BD (Construction)

∴ $\frac{AE}{EB} = \frac{AC}{CD}$ . . .(i) By B.P.T

ray CE || ray BD and AD is transversal

∴ ∠ ACE ≅ ∠ CDB . . .(ii) (Corresponding angles) 2

Now, BC as transversal

∠ ECB ≅ ∠ CBD . . .(iii) (Alternate angles)

But, ∠ACE ≅ ∠ECB . . .(iv) (Given)

In Δ CBD,

∴ ∠ CBD ≅ ∠ CDB [from (ii), (iii), (iv)]

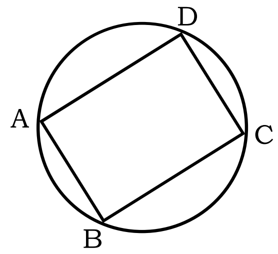
∴ seg CB ≅ seg CD . . .(v) (converse of isosceles triangle theorem)

∴ $\frac{AE}{EB} = \frac{CA}{CB}$ from (i) and (v)

(iii) Given : □ABCD is cyclic

To prove : ∠ A + ∠ C = 180°

Proof :



∠ A = $\frac{1}{2}$ m (arc BCD) ...(i)

∠ C = $\frac{1}{2}$ m (arc BAD) ...(ii)

} (Inscribed angle theorem) 2

Adding (i) and (ii)

∠ A + ∠ C = $\frac{1}{2}$ [m (arc BCD) + m (arc BAD)]

∴ ∠ A + ∠ C = $\frac{1}{2} \times 360$

∴ ∠ A + ∠ C = 180°

A.3.(B) Solve ANY TWO of the following :

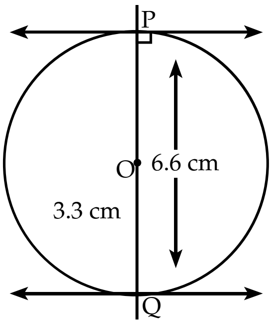
(i) **Analytical figure:**

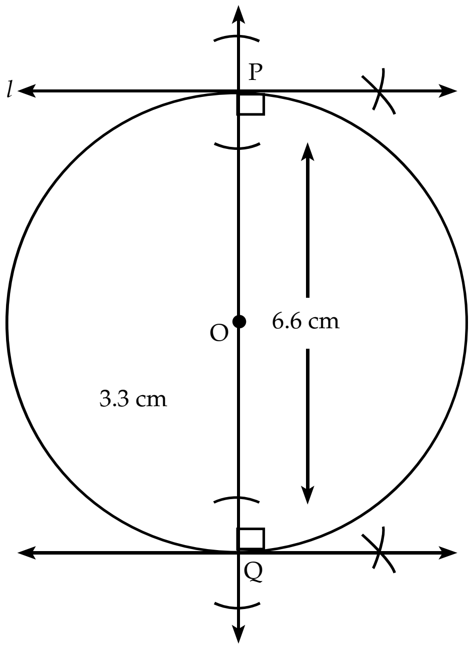
Radius = 3.3 cm (Given)

Chord = 6.6 cm (Given)

∴ Chord is twice of radius.

∴ Chord PQ is a diameter.





1 mark for drawing circle and diameter

2 marks for drawing tangents at P & Q.

(ii)	$A(3, 8) = (x_1, y_1)$ $B(-9, 3) = (x_2, y_2)$ Let point $P(0, a)$ be a point on Y-axis which divides seg AB in the ratio $m : n$. $P(0, a) = (x, y)$ By Section formula,	
	$x = \dfrac{mx_2 + nx_1}{m+n}$	½
	$0 = \dfrac{m \times (-9) + n(3)}{m+n}$	½
	$\therefore \quad 0 \times (m + n) = -9m + 3n$	
	$\therefore \quad 0 = -9m + 3n$	½
	$\therefore \quad 9m = 3n$ $\therefore \quad \dfrac{m}{n} = \dfrac{3}{9}$ $\therefore \quad \dfrac{m}{n} = \dfrac{1}{3}$ $\therefore \quad m : n = 1 : 3$	
	$\therefore$ Y-axis divides segment joining points A and B in the ratios 1 : 3	½
(iii)	LHS $= \sqrt{\sec^2\theta + \text{cosec}^2\theta}$	
	$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$	½
	$\left[\because 1 + \tan^2\theta = \sec^2\theta \quad 1 + \cot^2\theta = \text{cosec}^2\theta\right]$ $= \sqrt{\tan^2\theta + 2 + \cot^2\theta}$ $= \sqrt{\tan^2\theta + 2 \times 1 + \cot^2\theta}$	
	$= \sqrt{\tan^2\theta + 2 \times \tan\theta \times \cot\theta + \cot^2\theta}$	½
	$\left(\tan\theta = \dfrac{1}{\cot\theta} \quad \tan\theta \times \cot\theta = 1\right)$	

$= \sqrt{(\tan\theta + \cot\theta)^2}$ ½

$= \tan\theta + \cot\theta$ ½

$=$ R.H.S.

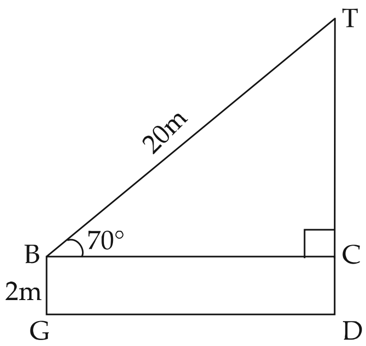
$\therefore \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

A.4. Solve ANY THREE of the following :

(i) GD is ground level.

BC is base of the ladder of the fire brigade van at a height of 2 m from ground level.

'T' is top of ladder of the fire brigads van at the maximum height ½



ĎTBC = 70° ...(Angle of elevation)

BT is the length of the ladder

BT = 20 m, BG = 2 m

$\square$BGDC is a rectangle ...(By definition) ½

BG = CD = 2 m ...(Opposite sides of a rectangle)

In $\triangle$BCT, ĎBCT = 90

$\therefore$ sin ĎTBC = $\frac{TC}{TB}$ ...(By definition) ½

$\therefore \sin 70° = \frac{TC}{20}$

$\therefore 0.94 = \frac{TC}{20}$

$\therefore TC = 0.94 \times 20$

$\therefore TC = 18.80$ m ½

TD = TC + CD ...(T – C – D) ½

$\therefore$ TD = 18.80 + 2

$\therefore$ TD = 20.80 m

y Other end of the ladder can reach 20.80 m above the ground ladder. ½

(ii)

For cylindrical wrapper,

Diameter = 14 mm ½

Radius (R) $\frac{14}{2}$ mm = 7 mm

Height (H) = 10 cm

i.e. H = 100 mm

For cylindrical tablet, ½

Radius (r) = 7 mm, Height (h) = 5 mm

Let 'N' number of tablets can be wrapped in the given wrapper.

$\therefore$ N × Volume of tablet = Volume of wrapper. ½

$\therefore$ $N \times \pi r^2 h = \pi R^2 H$ ½

$\therefore$ $N \times \pi \times 7 \times 7 \times 5 = \pi \times 7 \times 7 \times 100$

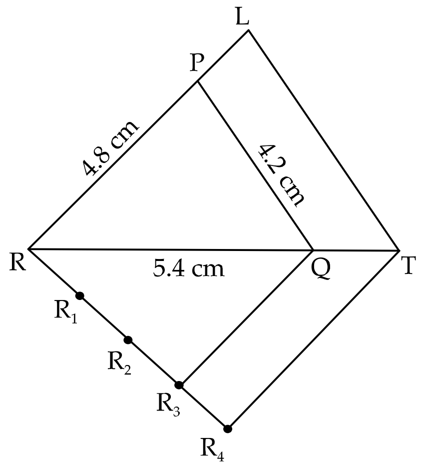
$\therefore$ $N = \frac{\pi \times 7 \times 7 \times 100}{\pi \times 7 \times 7 \times 5}$ ½

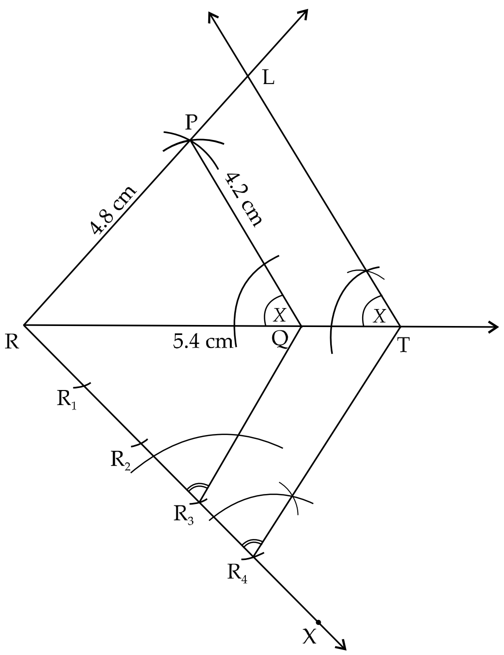
$\therefore$ N = 20

$\therefore$ **20 tablets can be packed in the given wrapper.** ½

(iii)

Analytical figure:



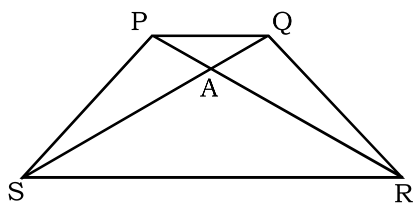


1 mark for ΔPQR

1 mark for constructing $\angle RR_3Q \cong \angle RR_4T$

1 mark for constructing $\angle RQP \cong \angle RTL$

(iv)



AR = 5AP ...(Given)

$\therefore \quad \frac{AR}{AP} = \frac{5}{1}$ ...(i) ½

AS = 5AQ ...(Given)

$\therefore \quad \frac{AS}{AQ} = \frac{5}{1}$ ...(ii) ½

In ASR and AQP,

$\frac{AR}{AP} = \frac{AS}{AQ}$ ...[From (i) and (ii)].

$\angle SAR \cong \angle QAP$ ...(Vertically opposite angles) 1

$\therefore$ ASR ~ AQP ...(By SAS Test of similarity)

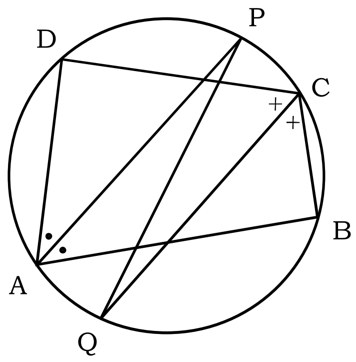
$\therefore \quad \frac{SR}{PQ} = \frac{AR}{AP}$ ...(c.s.s.t.) ½

$\therefore \quad \frac{SR}{PQ} = \frac{5}{1}$ ...[From (i)]

$\therefore$ SR = 5 PQ ½

A.5. Solve ANY ONE of the following :

(i) Proof :



$\angle DAP \cong \angle PAB$ [Ray AP bisects $\angle DAB$]

Let $m\angle DAP = m\angle PAB = x^\circ$ ...(i)

$\angle DCQ \cong \angle QCB$ [Ray CQ bisects $\angle DCB$]

Let $m\angle DCQ = m\angle QCB = y^\circ$ ...(ii) ½

$\square$ABCD is cyclic

$\therefore$ $\angle DAB + m\angle DCB = 180^\circ$

[Opposite angles of cyclic quadrilateral are supplementary] ½

$\therefore$ $\angle DAP + \angle PAB + \angle DCQ + \angle QCB = 180^\circ$

$\therefore$ $x + x + y + y = 180$ [From (i) and (ii)]

$\therefore$ $2x + 2y = 180$ ...(iii)

$m\angle DAP = \frac{1}{2}$ m(arc DP) [Inscribed angle theorem] ½

$\therefore$ $x = \frac{1}{2}$ m(arc DP) [From (i)]

$\therefore$ m(arc DP) $= 2x$ ...(iv)

$m\angle DCQ = \frac{1}{2}$ m(arc DAQ) [Inscribed angle theorem] ½

$\therefore$ $y = \frac{1}{2}$ m(arc DAQ) [From (ii)]

$\therefore$ m(arc DAQ) $= 2y$ ...(v) ½

$2x + 2y = 180$ ...(iii)

m(arc DP) $= 2x$ ...(iv)

m(arc DAQ) $= 2y$ ...(v)

Adding (iv) and (v),

m(arc DP) + m(arc DAQ) $= 2x + 2y$ ½

$\therefore$ m(arc PDQ) $= 2x + 2y$ [Arc addition property]

$\therefore$ m(arc PDQ) $= 180^\circ$ [From (iii)] ½

$\therefore$ Arc PDQ is a semicircle

$\therefore$ Seg PQ is the diameter of the circle ½

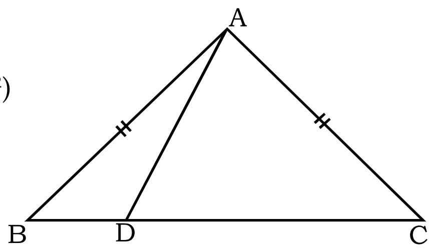
(ii) Construction : Draw seg AE $\perp$ side BC such that B – D – E - C ½

Proof : In ΔAEB, m$\angle$AEB = 90° [Construction]

$\therefore$ $AB^2 = AE^2 + BE^2$ ...(i) [By Pythagoras theorem] ½

In ΔAED, m$\angle$AED = 90° [Construction]

$\therefore$ $AD^2 = AE^2 + BE^2$ ...(ii) [By Pythagoras theorem] ½



Subtracting equation (ii) from (i),

$AB^2 - AD^2 = AE^2 + BE^2 - (AE^2 + DE^2)$ ½

$\therefore$ $AB^2 - AD^2 = AE^2 + BE^2 - AE^2 - DE^2$

$\therefore$ $AB^2 - AD^2 = BE^2 - DE^2$

$\therefore$ $AB^2 - AD^2 = (BE + DE)(BE - DE)$

$\therefore$ $AB^2 - AD^2 = (BE + DE) \times BD$ ...(iii) [B – D – E] ½

In ΔAEB and ΔAEC,

m$\angle$AEB = m $\angle$AEC = 90° [Construction]

Hypotenuse AB $\cong$ Hypotenuse AC [Given]

seg AE $\cong$ seg AE [Common side]

$\therefore$ ΔAEB $\cong$ ΔAEC [Hypotenuse-side theorem] 1

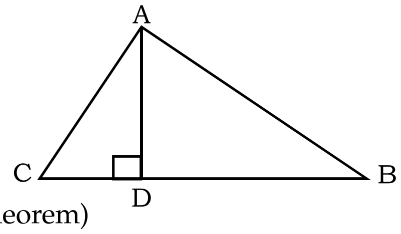
seg BE $\cong$ seg CE …(iv) [c.s.c.t.]

$AB^2 - AD^2 = (CE + DE) \times BD$ [from (iii) and (iv)]

$AB^2 - AD^2 = CD \times BD$ [C – E – D] ½

A.6. Solve ANY ONE of the following :

(i) To Prove : $2AB^2 = 2AC^2 + BC^2$



Proof :

DB = 3CD ...(i) (Given)

In ΔADB, $\angle$ADB = 90° (Given)

$\therefore$ $AB^2 = AD^2 + DB^2$ (By Pythagoras theorem) ½

$\therefore$ $AB^2 = AD^2 + (3CD)^2$ [From (i)]

$\therefore$ $AB^2 = AD^2 + 9CD^2$ ...(ii)

In ΔADC, $\angle$ADC = 90° ...(Given)

$\therefore$ $AC^2 = AD^2 + CD^2$ (By Pythagoras theorem) ½

$\therefore$ $AD^2 = AC^2 - CD^2$ ...(iii)

$AB^2 = AC^2 - CD^2 + 9CD^2$ [From (ii) and (iii) ½

$\therefore$ $AB^2 = AC^2 + 8CD^2$ ...(iv)

But BC = CD + DB ...[C - D - B]

$\therefore$ $BC = CD + 3CD$ ...[From (i)]

$\therefore$ $BC = 4CD$

$\therefore$ $CD = \frac{BC}{4}$ ...(v) ½

$\therefore$ $AB^2 = AC^2 + 8\left(\frac{BC}{4}\right)^2$ ...[From (iv) and (v)]

$\therefore$ $AB^2 = AC^2 + 8\ \frac{BC^2}{16}$ ½

$\therefore$ $AB^2 = AC^2 + \frac{BC^2}{2}$ ...[From (iv) and (v)]

$\therefore$ $\boxed{2AB^2 = 2AC^2 + BC^2}$ (Multiplying throughout by 2) ½

(ii) In Δ PQR

$\angle PQR = 90^\circ$ [Tangent Theorem]

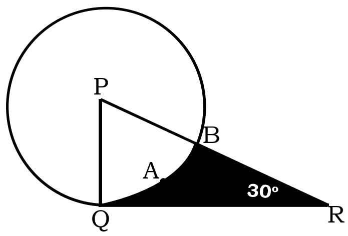
$\angle R = 30^\circ$ [Given]

$\therefore$ $\angle P = 60^\circ$ [Remaing angle]

$\therefore$ ΔPQR is $30^\circ - 60^\circ - 90^\circ$ triangle

$PQ = \frac{1}{2} PR$ ($30^\circ - 60^\circ - 90^\circ$ triangle theorem)

$= \frac{1}{2} \times 12$



$PQ = 6$ cm

$\therefore$ Radiam of the circle (r) = 6 cm

$QR = \frac{\sqrt{3}}{2} PR$

$= \frac{\sqrt{3}}{2} \times 12$

$\therefore$ $QR = 6\sqrt{3}$

$\therefore$ $A(\Delta PQR) = \frac{1}{2} QR \times PQ$

$= \frac{1}{2} \times 6\sqrt{3} \times 6$

$= 18\sqrt{\ } = 18 \times 1.73$

$= 31.14\ cm^2$ 1

Area of a sector $= \frac{\ }{360} \times \pi r^2$

	A(P–QAB) $= \frac{60}{360}\ 31.14 \times 6^2$	
	$= \frac{1}{6} \times 3\ 14 \times 6 \times 6 = 3.14 \times 6$	
	$= 18.84\ cm^2$	1
	Area of shaded region = A(PQR) – A(P–QAB)	
	= 31.14 – 18.84	
	$= 12.30\ cm^2$	1
	Area of the shaded region = $12.30\ cm^2$	
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